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Summary. — A new interpretation is given of Afred N. Whitehead's 1922 theory of gravity, which was considered a viable alternative to Einstein's theory until 1971. The strong equivalence principle is satisfied by this new interpretation. The history, motivation and pedagogical advantages underlying the new interpretation are discussed. This version of Whitehead's theory passes the gravimeter tests which the old version failed. Tests such as the Nordtvedt effect and the binary pulsar are considered here, but no final conclusions are given.

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1. – Introduction.

Alfred N. Whitehead's 1922 theory of gravity [1], as interpreted by Synge [2] in 1952, is a wonderfully simple model which accounts for all gravitational phenomena known to us prior to 1971 [3]. As a conceptual bridge from Newton's theory to Einstein's (or to the new interpretation presented below), it could be of great value if taken seriously. Whitehead's theory is analogous to the Lienard-Wiechert formulation of classical electrodynamics, the pedagogical attributes of which have been especially stressed by Feynman. In the first volume of his lecture series [4], Feynman put things strictly in terms of retarded interactions between point charges «to impress the reader with the beauty of nature, so to

(1) A. N. WHITEHEAD: *The Principle of Relativity* (Cambridge University Press, London, 1922).
(2) J. L. SYNGE: *Proc. R. Soc. London, Ser. A*, **211**, 303 (1952).
(3) C. M. WILL: *Astrophys. J.*, **169**, 141 (1971).
(4) R. P. FEYNMAN, R. B. LEIGHTON and M. L. SANDS: *The Feynman Lectures on Physics*, Vol. 1 (Addison-Wesley, Reading, Mass., 1964), p. 28-2.

speak, *i.e.* that it is possible to summarize all the fundamental knowledge on one page, with notations that he is now familiar with».

Of course, Einstein's theory is nonlinear and therefore not susceptible to such a simple formulation. Whitehead's theory is linear and takes the following form:

$$g_{\mu\nu} = \eta_{\mu\nu} + \sum_{i=1}^{N} h^i_{\mu\nu}. \tag{1}$$

This is the recipe for the metric tensor affecting a particle at a field point, due to N other source particles. The formulae for $h_{\mu\nu}$ will be given in sect. 2 and 3. Whitehead was a bit vague about these formulae, but Synge's interpretation provided the required specificity. My interpretation amends these formulae somewhat. Just as in classical electrodynamics, the summation in eq. (1) excludes effects of a point particle on itself (which would be infinite). Greek indices run from zero to three and η is the Minkowski tensor which has only diagonal elements $(-1, 1, 1, 1)$.

Whitehead's law of gravity is manifestly Lorentz covariant, rather than generally covariant. However, it could be written in generally covariant form if one wanted to go to the trouble [5].

In order to satisfy the weak equivalence principle [6], Whitehead's theory retains Einstein's geodesic equations of motion for material particles (commas denote partial differentiation and repeated indices are summed):

$$\frac{d^2x^\alpha}{ds^2} = -\frac{1}{2} g^{\alpha\mu}(g_{\mu\lambda,\beta} + g_{\mu\beta,\lambda} - g_{\beta\lambda,\mu}) \frac{dx^\lambda}{ds}\frac{dx^\beta}{ds}, \tag{2}$$

with $g^{\alpha\mu} g_{\alpha\lambda} = \delta^\mu_\lambda$,

$$g_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = -1. \tag{3}$$

In both Synge's interpretation and mine, gravity itself propagates along straight lines, with respect to the preferred reference frames. Synge interpreted the speed of gravitational propagation to be a universal constant, whereas I «allow» this speed to vary with an upper limit. Whether nature allows the new interpretation is the important point and is something which should be investigated. Unlike Synge's, my interpretation incorporates the strong equivalence principle, and therefore appears consistent with all known experiments (possible exceptions are discussed in appendix B).

The salient feature of Whitehead's theory, beyond its obvious simplicity, is

(5) C. M. WILL: *Theory and Experiment in Gravitational Physics* (Cambridge University Press, London, 1981), p. 17.

(6) S. WEINBERG: *Gravitation and Cosmology* (Wiley, New York, N. Y., 1972), Chapt. 3.

the algebraic form of $g_{\mu\nu}$: as will be seen, g_{ab} is positive-definite in gravitational fields of arbitrary strength (the letters a and b run from one to three). That is, for any three-vector $\boldsymbol{U}$,

$$g_{ab}U_aU_b \geqslant 0\,. \tag{4}$$

As was first pointed out by Hilbert (7), and as is evident from eq. (3) above, this positive-definiteness implies that $dx^0/ds \neq 0$ for material particles. Hence, a particle's proper time s can be considered a function of a coordinate time x^0. In Einstein's theory, however, there is no assurance that one can choose coordinates such that there is a universal time coordinate.

A few historical comments about Whitehead's theory are in order. In 1924, Sir Arthur Eddington showed (8) that the field of a static point mass is identical in Einstein's and Whitehead's theories. In so doing, Eddington stumbled upon a previously unknown form of the Schwarzschild metric, a form which not only served (9) in 1963 as a model for the construction of the Kerr metric (describing a spinning point-mass in Einstein's theory), but which also contains no artificial «Schwarzschild singularity». This form was discovered a full 36 years before Kruskal found his famous singularity-free coordinate system (10) in 1960. Thus, far from being a useless relic of an obscure philosopher, Whitehead's theory has actually contributed significantly to the progress of gravitational research during the past thirty years. The chronology shows that Whitehead's theory was not designed to agree with the Schwarzschild metric, and indeed this agreement is quite a coincidence. The theory was viewed, up to 1971, as a serious competitor with Einstein's general relativity, since it passed the classical experimental tests: redshift, perihelion shift, light deflection and time delay (3).

In 1971, Will argued that (3), according to Synge's interpretation, the effect of the rest of our galaxy cannot merely be transformed away (as it can in Einstein's theory), and must give rise to an apparent anisotropy in the Newtonian gravity constant. Because the anisotropy did not show up in very sensitive gravimeter experiments, Whitehead's theory was declared defunct. Although Will used what is now an outdated Keplerian model of our galaxy, with no «dark matter» lurking about, his analysis did not depend heavily on what galactic model was used. The null gravimeter experiments lend strong support to the strong equivalence principle, according to which uniform gravitational fields can be eliminated by a coordinate transformation.

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(7) D. HILBERT: *Gesammelte Abhandlungen*, Vol. 3 (Springer, Berlin, 1970), p. 271.
(8) A. S. EDDINGTON: *Nature (London)*, **113**, 192 (1924).
(9) R. ADLER, M. BAZIN and M. SCHIFFER: *Introduction to General Relativity* (McGraw-Hill, New York, N.Y., 1975), p. 238. A time reversal reconciles the formulae.
(10) C. W. MISNER, K. S. THORNE and J. A. WHEELER: *Gravitation* (Freeman, San Francisco, Cal., 1973), p. 828; see also ref. (6), p. 208.

Aesthetically, the strong equivalence principle is an appealing feature of any gravity theory, and it should be no surprise that experiments support this view. That is why reinterpretation of Whitehead's theory, so as to incorporate the strong equivalence principle, has been debated (11) as recently as 1975, and why the debate is not yet over.

Incidentally, we will be concerned throughout this article with uncharged point particles. Neverheless, the extension to smoothly distributed neutral matter (12), and thence to charged matter (2), is not a great problem. Likewise, the cosmologies allowed by Whitehead's theory can be examined elsewhere (13).

2. – Synge's interpretation.

Synge's interpretation (2) of Whitehead's theory will now be written in three different ways. They are completely equivalent and might best be described as «four-dimensional», «three-dimensional» and «five-dimensional». The first two approaches are often used in classical electrodynamics (14), but the simplicity and utility of the five-dimensional notation seem to have gone unnoticed. The four-dimensional version is the one which Synge took as his starting point. The five-dimensional notation will be used in the next section, to present a new interpretation of Whitehead's theory.

The four-dimensional version is simply this:

$$h_{\mu\nu} = \left[\frac{2MG\,R_\mu R_\nu}{(R\cdot V)^3}\right]_{\text{ret}} . \tag{5}$$

G is Newton's gravity constant and M is the constant mass of a source-particle:

$$\begin{cases} V^\mu \equiv \dfrac{\mathrm{d}x^\mu}{\mathrm{d}p}, \qquad V\cdot V = -1, \qquad R\cdot R = 0, \\ R^\mu \equiv [x^\mu]_{\text{source}} - [x^\mu]_{\text{field}} \qquad \text{and} \qquad W^\mu \equiv \dfrac{\mathrm{d}^2 x^\mu}{\mathrm{d}p^2}. \end{cases} \tag{6}$$

Of course, a dot indicates contraction with the Minkowski tensor η, which is also used to raise and lower indices of V, R and W (W is defined for later use). The parameter p is to be distinguished, in Synge's interpretation, from the proper

(11) C. C. CHIANG and V. H. HAMITY: *Lett. Nuovo Cimento*, **13**, 471 (1975). These authors obtain the results of ref. (3) in a much simpler way.
(12) C. B. RAYNER: *Proc. R. Soc. London, Ser. A*, **222**, 509 (1954).
(13) J. D. NORTH: *The Measure of the Universe* (Oxford University Press, London, 1965), p. 194.
(14) J. D. JACKSON: *Classical Electrodynamics* (Wiley, New York, N.Y., 1975), p. 657.

time s which appears above in the equations of motion. It should be clear that, according to eqs. (6), gravity propagates with a speed equal to unity and that particles have speeds less than unity. This speed limit is consistent with eqs. (1), (3) and (5), according to which

$$\eta_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = -1 - \frac{dx^\mu}{ds}\frac{dx^\nu}{ds}\sum_{i=1}^{N} h^i_{\mu\nu} < 0 . \tag{7}$$

Equation (5) will now be written in explicit three-dimensional language. The same thing can be, and has been, done with the equations of motion [15]:

$$h_{00} = \left[\frac{2MG\,r^2}{(r + \boldsymbol{r}\cdot\boldsymbol{v})^3}(1-v^2)^{3/2}\right]_{\text{ret}}, \tag{8}$$

$$h_{ab} = \left[\frac{2MG\,r_a r_b}{(r + \boldsymbol{r}\cdot\boldsymbol{v})^3}(1-v^2)^{3/2}\right]_{\text{ret}}, \tag{9}$$

$$h_{0a} = \left[\frac{2MG\,r_a r}{(r + \boldsymbol{r}\cdot\boldsymbol{v})^3}(1-v^2)^{3/2}\right]_{\text{ret}}, \tag{10}$$

$$v_a \equiv \frac{dx^a}{dx^0}, \quad r_a \equiv [x^a]_{\text{source}} - [x^a]_{\text{field}}, \quad r \equiv \sqrt{\boldsymbol{r}\cdot\boldsymbol{r}} \quad \text{and} \quad v \equiv \sqrt{\boldsymbol{v}\cdot\boldsymbol{v}} . \tag{11}$$

Notice that, since $v<1$, g_{ab} is positive-definite as claimed in the introduction.

If $v=0$, the static limit of eqs. (1) and (5) is obvious, for the case of a single source-particle:

$$g_{00} = -1 + \frac{2MG}{r}, \quad g_{0a} = -\frac{2MG\,x^a}{r^2} \quad \text{and} \quad g_{ab} = \delta_{ab} + \frac{2MG\,x^a x^b}{r^3} . \tag{12}$$

As discussed above, this is none other than the Schwarzschild metric [9]. It is free of artificial singularities. Note that the determinant of the metric tensor equals negative one. Furthermore, it is easy to show that eqs. (12) represent a white hole rather than a black hole. Light travels along null geodesics so:

$$0 = g_{\mu\nu}\frac{dx^\mu}{dx^0}\frac{dx^\nu}{dx^0} = g_{00} + 2g_{0a}v_a + g_{ab}v_a v_b = \tag{13}$$
$$= -1 + \frac{2MG}{r} + \frac{4MG(\boldsymbol{r}\cdot\boldsymbol{v})}{r^2} + v^2 + \frac{2MG(\boldsymbol{r}\cdot\boldsymbol{v})^2}{r^3} .$$

Therefore,

$$\boldsymbol{r}\cdot\boldsymbol{v} < 0 \qquad \text{if } -1 + \frac{2MG}{r} > 0 . \tag{14}$$

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[15] See ref. [6], p. 212.

Ergo, at the Schwarzschild radius $r=2MG$, light must be outgoing, as was to be proved.

Now, Synge's interpretation will be written one last way: in five-dimensional form. We begin by defining [16] a «world function»

$$\Omega(x^0, x^1, x^2, x^3, p) \equiv \frac{1}{2}\eta_{\mu\nu}[x^\mu - x^\mu(p)][x^\nu - x^\nu(p)]. \tag{15}$$

Thus, according to eqs. (5) and (6),

$$h_{\mu\nu} = \frac{2MG\Omega_{,\mu}\Omega_{,\nu}}{\Omega^3_{,p}} \tag{16}$$

if

$$\Omega = 0 \quad \text{and} \quad \eta^{\mu\nu}\Omega_{,\mu,p}\Omega_{,\nu,p} = -1. \tag{17}$$

To demonstrate the usefulness of this formulation, it will be employed here to differentiate $h_{\mu\nu}$, as required by eq. (2). Then the four-dimensional format will be recovered. Notice that the left-hand side of eq. (16) is a functional of four coordinates, whereas the right-hand side is a functional of five variables. Hence (see appendix A), we differentiate as follows:

$$h_{\mu\nu,\lambda} = 2MG\left[\left(\frac{\Omega_{,\mu}\Omega_{,\nu}}{\Omega^3_{,p}}\right)_{,\lambda} - \frac{\Omega_{,\lambda}}{\Omega_{,p}}\left(\frac{\Omega_{,\mu}\Omega_{,\nu}}{\Omega^3_{,p}}\right)_{,p}\right] = \tag{18}$$

$$= 2MG\left[\frac{\Omega_{,\mu,\lambda}\Omega_{,\nu} + \Omega_{,\nu,\lambda}\Omega_{,\mu}}{\Omega^3_{,p}} - \frac{3\Omega_{,\mu}\Omega_{,\nu}\Omega_{,p,\lambda}}{\Omega^4_{,p}} - \right.$$

$$\left. - \frac{\Omega_{,\lambda}}{\Omega_{,p}}\left(\frac{\Omega_{,\mu,p}\Omega_{,\nu} + \Omega_{,\mu}\Omega_{,\nu,p}}{\Omega^3_{,p}} - \frac{3\Omega_{,\mu}\Omega_{,\nu}\Omega_{,p,p}}{\Omega^4_{,p}}\right)\right].$$

So

$$h_{\mu\nu,\lambda} = 2MG\left[-\frac{\eta_{\mu\lambda}R_\nu + \eta_{\nu\lambda}R_\mu}{(R\cdot V)^3} + \frac{3R_\mu R_\nu V_\lambda}{(R\cdot V)^4} + \right. \tag{19}$$

$$\left. + \frac{R_\lambda(V_\mu R_\nu + R_\mu V_\nu)}{(R\cdot V)^4} + \frac{3R_\lambda R_\mu R_\nu}{(R\cdot V)^5} - \frac{3R_\lambda R_\mu R_\nu (R\cdot W)}{(R\cdot V)^5}\right]_{\text{ret}}.$$

The last term in eq. (19) is the radiation term, as it is the only term with a $1/r$

([16]) J. L. SYNGE and B. S. DE WITT: in *Relativity, Groups, and Topology*, edited by C. DE WITT and B. DE WITT (Blackie, London, 1964), p. 37 and p. 735.

dependence. Note that this sort of technique could easily be used in differenting the Lienard-Wiechert potentials, to obtain the retarded field strengths of classical electrodynamics.

3. – The new interpretation.

In five-dimensional notation, the new interpretation of Whitehead's theory is

$$\Omega(x^0, x^1, x^2, x^3, s) \equiv \frac{1}{2}[g_{\mu\nu}(s)][x^\mu - x^\mu(s)][x^\nu - x^\nu(s)] , \tag{20}$$

$$h_{\mu\nu} = \frac{2MG\Omega_{,\mu}\Omega_{,\nu}}{\Omega^3_{,s}} \tag{21}$$

if

$$\Omega = 0 \quad \text{and} \quad \Omega_{,s} > 0 . \tag{22}$$

The parameter s is the proper time which appears in the equations of motion; there is no extra parameter p (critics of Whitehead's theory have found this extra parameter to be particularly distasteful ([17])). Of course, it is clear from the previous section that the dimensionality of our notation could be changed at will.

A most important feature of eqs. (1), (20), (21) and (22) is their implication that particles interact gravitationally over timelike intervals, *i.e.* that the speed of gravitational propagation is less than unity. This is evident since

$$0 = [g_{\mu\nu}(s)][x^\mu - x^\nu(s)][x^\nu - x^\nu(s)] \tag{23}$$

and therefore

$$0 > \eta_{\mu\nu}[x^\mu - x^\mu(s)][x^\nu - x^\nu(s)] . \tag{24}$$

Consequently, if gravitational interactions appear retarded in one Lorentz frame, then they will appear retarded in any other. No advanced effects will enter in.

Another notable feature of this new interpretation is that, in order to calculate $h_{\mu\nu}$ due to a particle at a retarded position, we need to know not only the retarded position and velocity of this particle, but also the retarded values of the metric tensor and its first derivatives. This feature is analogous to the fact that, in Einstein's theory, one solves a Cauchy initial-value problem by specifying not

([17]) A. SCHILD: in *Evidence for Gravitational Theories*, edited by C. MOLLER (Academic Press, New York, N. Y., 1962), p. 82.

only initial distributions of mass and velocity, but also initial values of the metric tensor and its first «time» derivatives [18].

It is easy to see that, for a static point-particle subject to no external influences, this new interpretation yields the Schwarzschild metric given by eqs. (12) above. Equations (12) specifically rule out black holes and thus far their existence has not been experimentally proved [19].

As for the strong equivalence principle, let us begin by stating very precisely what it means [20]. Suppose the difference $g_{\mu\nu} - \eta_{\mu\nu}$ takes a virtually constant value $K_{\mu\nu}$ on a surface far from an enclosed gravitational system. The principle is satisfied if the law of gravity is covariant with respect to a coordinate transformation which gets rid of $K_{\mu\nu}$. The principle is not satisfied by the old interpretation of Whitehead's theory, but it is by the new one.

To see that this is true, consider the linear coordinate transformation

$$x^\alpha = \Lambda^\alpha_\mu x^{\mu'} . \tag{25}$$

The Λ's are constants subject to the condition

$$\eta_{\alpha\beta} = (\eta_{\mu\nu} + K_{\mu\nu}) \Lambda^\mu_\alpha \Lambda^\nu_\beta . \tag{26}$$

Note that this merely reduces to a Lorentz transformation [21] if $K_{\mu\nu}$ is zero. Anyway, eq. (25) transforms away $K_{\mu\nu}$ on an enclosing surface, according to eq. (1) and the transformation law [6]

$$g'_{\alpha\beta} = g_{\mu\nu} \frac{\partial x^\mu}{\partial x^{\alpha'}} \frac{\partial x^\nu}{\partial x^{\beta'}} . \tag{27}$$

But, is the law of gravity covariant under this transformation? Yes. The beautiful thing about eqs. (20)-(22) is that they are covariant with respect to any linear coordinate transformation whatsoever, including that given by eqs. (25) and (26).

4. – Remarks.

In discussing the strong equivalence principle (SEP), it has been assumed that the metric tensor takes a virtually constant value on a surface far from an enclosed gravitating system. Indeed, this is true for a surface which contains our solar system and is immersed in the Milky Way. However, one might want to

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([18]) See ref. ([6]), p. 163.
([19]) S. W. HAWKING: *A Brief History of Time* (Bantam, New York, N. Y., 1988), p. 94.
([20]) See ref. ([5]), subsect. 3·3.
([21]) See ref. ([6]), p. 26.

take a broader view of SEP and also consider enclosing surfaces on which the first (or higher) partial derivatives of the metric tensor are nonzero and virtually constant. Although Einstein's theory satisfies this broad view of SEP in the case of first partial derivatives (higher derivatives of the metric tensor can only be transformed away if space-time is flat), there does not appear to be a compelling reason for requiring the same of Whitehead's theory.

An argument in favor of Einstein's theory is that it cannot help but produce an inverse-square law in the Newtonian limit. Although Whitehead's theory is designed to have this limit, it should be emphasized that there is nothing wrong with preferring an r^{-2} to an $r^{-2.1}$ force law. Only the former yields Kepler's beautiful laws of conic motion.

Furthermore, some might argue that Einstein's theory is better because it has no preferred reference frames. However, the fact is that there are indeed such frames: those in which the metric is asymptotically Minkowskian, or for which one can write down simple coordinate conditions.

The preference shown in this article for retarded over advanced interactions may at first seem rather arbitrary. Nevertheless, it is required by common sense. Combinations of advanced and retarded potentials make perfect sense when dealing with a Cauchy initial-value problem, such as is presented by Maxwell's equations in classical electrodynamics. These combinations then merely correspond to different sorts of initial data. However, when dealing with a theory of direct particle interactions that cannot be described by field equations, the initial-value problem is very different and a knowledge of the future must not be necessary to predict that future.

Unlike Synge's interpretation, the new interpretation may entail discontinuities of the metric tensor, when extremely strong gravitational forces are involved. Without getting into details, it should suffice to say here that these discontinuities need not cause a particle's three-dimensional velocity to change discontinuously. Rather, the particle's proper time s must merely be renormalized according to eq. (3) above.

Finally, in all honesty, it should be pointed out that eq. (21) does seem arbitrary in one minor respect. There is one other formula for «$h_{\mu\nu}$» which satisfies the requirements discussed in this article, but which definitely violates experimental parameters. It is

$$h_{\mu\nu} = \frac{2MG\,\Omega_{,\mu,s}\Omega_{,\nu,s}}{\Omega_{,s}}. \tag{28}$$

I do not know why nature would have a preference for eq. (21) over eq. (28) and, as Newton might say, I frame no hypothesis.

* * *

I would like to acknowledge the assistance of many physics professors, whose open doors and ready advice are much appreciated.

Appendix A

In sect. 2, eq. (16) was differentiated to yield eq. (18), using the five-dimensional notation. This procedure can be clarified by considering a simple two-dimensional example. Suppose that

$$f(x) = F(x, y) \qquad \text{if } G(x, y) = 0 . \tag{A.1}$$

Thus, for any parameter u,

$$\frac{df}{du} = \frac{\partial F}{\partial x}\frac{dx}{du} + \frac{\partial F}{\partial y}\frac{dy}{du} \quad \text{and} \quad \frac{dG}{du} = \frac{\partial G}{\partial x}\frac{dx}{du} + \frac{\partial G}{\partial y}\frac{dy}{du} = 0 . \tag{A.2}$$

So,

$$\frac{df}{du} = \frac{\partial F}{\partial x}\frac{dx}{du} + \frac{\partial F}{\partial y}\left[-\frac{\partial G}{\partial x}\frac{dx}{du}\bigg/\frac{\partial G}{\partial y}\right]. \tag{A.3}$$

Setting $u = x$,

$$\frac{df}{dx} = \frac{\partial F}{\partial x} - \frac{\partial F}{\partial x}\frac{\partial G}{\partial x}\bigg/\frac{\partial G}{\partial y} . \tag{A.4}$$

The right side of eq. (A.4) is exactly analogous to the first bracketed term in eq. (18).

Appendix B

The purpose of this appendix is to elaborate on whether Whitehead's theory of gravity, as interpreted in sect. 3 above, is consistent with all known experiments. Although no final conclusion will be reached here, it is hoped that this discussion will provide a basis for further study.

An important tool for analyzing any metric theory of gravity is its post-Newtonian approximation. In the context of Synge's interpretation, this has already been derived by Clark [22]. For the record, the post-Newtonian approximation arising from the new interpretation will now be given. Proceeding along exactly the same lines as Clark, I find the following result for the metric tensor affecting the n-th particle in a system of N particles:

$$(r_a)_{ni} \equiv x_i^a - x_n^a , \tag{B.1}$$

$$g_{ab}^{(n)} = \delta_{ab} + 2G \sum_{i \neq n}^{N} \frac{M_i (r_a)_{in} (r_b)_{in}}{(r_{in})^3} , \tag{B.2}$$

$$g_{0a}^{(n)} = -2G \sum_{i \neq n}^{N} \frac{M_i}{(r_{in})^2}\left[(r_a)_{in} + r_{in}(v_a)_i + (r_a)_{in}\left(\frac{\boldsymbol{v}_i \cdot \boldsymbol{r}_{in}}{r_{in}}\right)\right], \tag{B.3}$$

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[22] G. L. Clark: *Proc. R. Soc. Edinburgh, Sect. A*, **64**, 49 (1954).

$$\text{(B.4)}\qquad g_{00}^{(n)} = -1 + 2G\sum_{i\neq n}^{N}\frac{M_i}{r_{in}}\left[1 + 2\left(\frac{\boldsymbol{v}_i\cdot\boldsymbol{r}_{in}}{r_{in}}\right) + (v_i)^2 - \frac{5}{2}(\boldsymbol{r}_{in}\cdot\boldsymbol{a}_i) + \frac{1}{2}\left(\frac{\boldsymbol{r}_{in}\cdot\boldsymbol{v}_i}{r_{in}}\right)^2\right] - 2G^2\sum_{i\neq n}^{N}\sum_{p\neq i}^{N}\frac{M_iM_p}{r_{in}r_{ip}}\left[2 + \left(\frac{\boldsymbol{r}_{in}\cdot\boldsymbol{r}_{ip}}{r_{in}r_{ip}}\right)^2\right].$$

Note that these equations would be identical to those of Clark if the term containing the double summation were omitted from eq. (B.4).

In order to compare this post-Newtonian approximation with those of other theories and with experiments, it would be convenient if we could perform a coordinate transformation that puts these equations into the PPN form developed by Will and Nordtvedt ([23,24]). This is what Will did in 1971 with Clark's equations ([3]). However, if one applies to eqs. (B.1)-(B.4) the gauge transformation which Will used in 1971 and performs the calculations as Will did, one finds that the doubly summed term in eq. (B.4) leads to a term which is not included in the PPN formalism. That is, the new interpretation of Whitehead's theory does not appear to fit into the PPN scheme.

As discussed in the introduction, the new interpretation evidently satisfies the four «classical» tests. Similarly, it is clear from the Lorentz covariance of the new interpretation that it excludes the possibility of preferred-frame effects ([23]). The analysis of SEP given in sect. 3 proves that the new interpretation is devoid of preferred location effects (of the sort which brought the downfall of Synge's interpretation). The Nordtvedt effect ([23]) is a comparatively complicated issue and it remains an important test which Whitehead's theory must pass. The same is true of the binary pulsar PSR 1913 + 16, for which the post-Newtonian approximation is not adequate to represent energy lost by gravitational radiation ([25]). The binary pulsar presents the additional problems of determining whether highly compact, spinning masses can be treated as point particles, and to what degree these point particles experience radiation-reaction forces. There are no obvious solutions to these problems even in the context of Einstein's theory ([26]).

Regardless of whether Whitehead's theory proves to be more or less correct than Einstein's theory in the experimental domain, perhaps this article will encourage a more serious analysis of the importance of Hilbert's inequality (eq. (4) in the introduction) in the context of Einstein's theory. After all, that inequality is what motivated this article.

([23]) See ref. ([5]), p. 10.
([24]) See ref. ([5]), p. 96.
([25]) See ref. ([5]), p. 90.
([26]) See ref. ([5]), p. 267 and p. 239.

Note added in proof.

It has recently come to my attention that eq. (21) above can be generalized by putting a factor of $\Omega^{K}_{,s,s}$ on the right-hand side. If this «Will» factor equals unity ($K = 0$), then the Nordtvedt parameter equals four-fifths ($\eta = 0.8$), and I am very grateful to Prof. C. M.

Will for proving this to me. I find that $K = 0.6$ if $\eta = 0$. That is, the Will factor must equal the tenth root of the sixth power of $\Omega_{,s,s}$ if massive spheres are to pursue geodesic trajectories. Given that $K = 0.6$ there appears an attractive $r^{-1.4}$ force which could be significant on a galactic distance scale.

● RIASSUNTO (*)

Si dà una nuova interpretazione della teoria di Alfred N. Whitehead del 1922 sulla gravità, che è stata considerata una valida alternativa alla teoria di Einstein fino al 1971. Il principio di forte equivalenza è soddisfatto da questa nuova interpretazione. Si discutono la storia, le ragioni e i vantaggi pedagogici che sottolineano la nuova interpretazione. Questa versione della teoria di Whitehead passa i test del gravimetro mancati dalla vecchia versione. Si considerano qui test come l'effetto di Nordtvedt e il pulsar binario ma non si danno conclusioni finali.

(*) *Traduzione a cura della Redazione.*

Новая интерпретация теории Уайтхеда.

Резюме (*). — Предлагается новая интерпретация теории гравитации Альфреда Н. Уайтхеда, сформулированной в 1922г., которая рассматривалась, как альтернативная теории Эйнштейна до 1971г. Эта новая интерпретация позволяет удовлетворить сильному принципу эквивалентности. Обсуждаются история, мотивация и педагогические преимущества новой интерпретации. Предложенный вариант теории Уайтхеда удовлетворяет гравиметрическим исследованиям, которым не удовлетворял старый вариант теории. В этой работе рассматриваются такие тесты, как эффект Нордведа и бинарный пульсар, однако не предлагаются окончательные выводы.

(*) *Переведено редакцией.*